Determination of the velocity of an emitter in spaces with affine connections and metrics

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Abstract

Doppler effect and Hubble effect in different models of space-time related to the space-time velocity of an observer are considered. The Doppler effect and Doppler shift frequency parameter are connected with the kinematic characteristics of the relative velocity of the emitter. The Hubble effect and Hubble shift frequency parameter are considered in analogous way. It is shown that by the use of the variation of the shift frequency parameter during a time period, considered locally in the proper frame of reference of an observer, one can directly determine the radial (centrifugal, centripetal) relative velocity and acceleration as well as the

tangential (Coriolis) relative velocity and acceleration of an astronomical object moving relatively to the observer. All results are obtained on purely kinematic basis without taking into account the dynamic reasons for the considered effect.

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1 Introduction

1. Modern problems of relativistic astrophysics as well as of relativistic physics (dark matter, dark energy, evolution of the universe, measurement of velocities of moving objects etc.) are related to the propagation of signals in space or in space-time [1], [2]. The basis of experimental data received as results of observations of the Doppler effect or of the Hubble effect gives rise to theoretical considerations about the theoretical status of effects related to detecting signals from emitters moving relatively to observers carrying detectors in their laboratories.

2. All considerations related to the relative motions of objects with respect to each other are made on the basis of the notions of relative velocity and relative acceleration. The relative velocity is usually considered as radial (centrifugal, centripetal) velocity and tangential (transversal, Coriolis) velocity. The radial velocity is one of the most important concept in astronomy and "a quantity whose precision of determination has improved significantly in recent years. Its measuring is generally understood as the object's motion along the line of sight, a quantity normally deduced from observed spectral-line displacements, interpreted as Doppler shifts." [3]

The naive concept of radial velocity from point of view of different theoretical models as the line-of-sight component of the stellar velocity vector measured by the Doppler shift of the spectral lines is for too simplistic when aiming at accuracies much better than 1 $km.sec^{-1}$.

The need for a stringent definition of radial velocity leads to a definition of this notion accepted by the 24^{th} General Assembly of the International Astronomical Union in year 2000 as a velocity of objects related to the s.c. barycentric radial velocity measure, defined in a Barycentric Celestial Reference System (BCRS) for accurate modelling of motions and events within the solar system. This system (BCRS) of co-ordinates interpreted as frame of reference serves as a quasi-Euclidean reference frame for the motions of nearby stars and more distance objects. The radial velocity could be measured by the use of two different methods:

(a) Spectroscopic method related to the Doppler effect. This method leads to the notion of spectrometric radial velocity.

Since the recently generally accepted theories of gravitation have no adequate formalism for description of the transversal Doppler effect, the notion of tangential (transversal, Coriolis) velocity is not uniquely defined for astronomical purposes. A spectroscopic line shift measurement, considered until now, is

equivalent to a direct comparison of the proper time scale at the emitter and the observer. The reason for this is the existence of special and general relativity theories considering the relations between different co-ordinate and proper times. For more complicated accelerated motions between observer and emitter (source) the relation between their proper time scales is not uniquely determined because the notions related to their definitions are defined, in general, as quantities not covariant or form invariant with resect to the change of their frames of reference [3].

(b) Geometric method related to the change of lengths and angles between an observer and moving objects. This method leads to the notions of kinematic radial (centrifugal, centripetal) velocity and kinematic tangential (transversal, Coriolis) velocity. The difficulties for the use of this method appear in astronomy when the relative velocity of objects at very large distances from Earth have to be measured.

The notion of astrometric radial velocity refers to the variation of the co-ordinates of the source (emitter), and therefore, depends on the chosen co-ordinate system and time scale. By contrast, the outcome of spectroscopic observation is a directly measurable quantity and, therefore, independent of co-ordinate systems [3].

3. Therefore, there are two different approaches for measuring radial velocities of cosmic objects: a covariant approach related to proper times (spectroscopic method) and a co-ordinate approach related to time and distances in a co-ordinate system identified as the proper frame of reference of the Sun (geometric method).

The covariant method is then specialized for the defined co-ordinate system. This leads to difficulties related to the different definitions of spectroscopic and kinematic quantities which are not directly connected to the real measurement of the radial or tangential velocities of astronomical objects. The comparison between the co-ordinate quantities and the spectroscopic data leads to introduction of notions such as "barycentric radial-velocity measure" and "astrometric radial velocity", where the notion of real radial velocity is avoided. The reason for the introduction of the above notions is the lack of covariant method for describing, on the one side, the relative velocities and accelerations and, on the other side, the lack of relations between these kinematic characteristics and their corresponding Doppler shifts and Hubble shifts. Recently, it has been shown that the introduced in $(\overline{L}_n,g)$-spaces kinematic characteristics related to relative velocities and relative accelerations could be in simple way expressed in terms of radial (centrifugal, centripetal) and tangential (transversal, Coriolis) velocities and accelerations [8]. On the other side, these kinematic characteristics could be related to an observer. The question arises how could one observer find his own space-time velocity and space-time acceleration with respect to an emitter (source) if the spectral data corresponding to the Doppler shifts and Hubble shifts are available. These velocity and acceleration could be considered as the velocity and acceleration of the emitter from point of view of the observer. This could be done because the observer, at first, determines the velocity and acceleration of the emitter from his proper frame of reference on the basis of the

shifts of the signals of the emitter, measured in observer's frame of reference, and then, on the basis of the relations between these shifts and the motion of the observer in space-time, the observer could find his space-time velocity and acceleration.

4. In the classical (non-quantum) field theories different models of space-time have been used for describing the physical phenomena and their evolution. The 3-dimensional Euclidean space E_3 is the physical space used as the space basis of classical mechanics [4]. The 4-dimensional (flat) Minkowskian space $\overline{M}_4$ is used as the model of space-time in special relativity [5]. The (pseudo) Riemannian spaces V_4 without torsion are considered as models of space-time in general relativity [6], [7]. In theoretical gravitational physics (pseudo) Riemannian spaces without torsion as well as (pseudo) Riemannian spaces U_4 with torsion are proposed as space-time grounds for new gravitational theories. To the most sophisticated models of space-time belong the spaces with one affine connection and metrics [(L_n, g)-spaces] and the spaces with affine connections and metrics [$(\overline{L}_n, g)$-spaces].

The spaces with one affine connection and metrics [(L_n, g)-spaces] have affine connections whose components differ only by sign for contravariant and covariant tensor fields over a differentiable manifold M with dim $M = n$.

The spaces with affine connections and metrics have affine connections whose components differ not only by sign for contravariant and covariant tensor fields over a differentiable manifold M with dim $M = n$.

3. Recently, it has been shown [8], [9] that every differentiable manifold M ($dim M = n$) with affine connections and metrics [$(\overline{L}_n, g)$-spaces] [10] could be used as a model of space-time for the following reasons:

- The equivalence principle (related to the vanishing of the components of an affine connection at a point or on a curve) holds in $(\overline{L}_n, g)$-spaces [11]÷[13].

- $(\overline{L}_n, g)$-spaces have structures similar to these in (pseudo) Riemannian spaces without torsion [V_n-spaces] allowing for description of dynamic systems and the gravitational interaction [9].

- Fermi-Walker transports and conformal transports exist in $(\overline{L}_n, g)$-spaces as generalizations of these types of transports in V_n-spaces [14], [15].

- A Lorentz basis and a light cone could not be deformed in $(\overline{L}_n, g)$-spaces as it is the case in V_n-spaces.

- All kinematic characteristics related to the notions of relative velocity and of relative acceleration could be worked out in $(\overline{L}_n, g)$-spaces without changing their physical interpretations in V_n-spaces [9], [16]÷[19].

- $(\overline{L}_n, g)$-spaces include all types of spaces with affine connections and metrics used until now as models of space-time.

4. If a $(\overline{L}_n, g)$-space could be used as a model of space or of space-time the question arises how signals could propagate in a space-time described by a $(\overline{L}_n, g)$-space. The answer of this question has been given in [19], [20], and [21]. By that the signals are described by means of null (isotropic) vector fields. A signal could be defined as a periodical process transferred by an emitter and received by an observer (detector) [20]. A wave front could be considered as a signal characterized by its wave vector (null vector) as it is the case in the geometrical optics in a V_n-space [22]. All results are obtained on *purely kinematic basis* (s. [19], [23], [24]) without taking into account the dynamic reasons for the considered effect.

On the basis of the general results in the previous papers we can draw a rough scheme of the relations between the kinematic characteristics of the relative velocity and relative acceleration on the one side, and the Doppler effect and the Hubble effect on the other. Here the following abbreviations are used:

CM - classical mechanics
SRT - special relativity theory
GRT - general relativity theory
CRT - classical relativity theory [8], [9].

5. The considerations of the Doppler effect and of the Hubble effect show that the Doppler effect is derived in the physical theories (with exception of general relativity) as a result of the relative motion of an observer and an emitter, sending signals to the observer, from point of view of the proper frame of reference of the observer and its relations to the proper frame of reference of the emitter. On the other side, the Hubble effect could be considered as a result of the Doppler effect and the Hubble law assumed to be valid in the corresponding physical theory. In a rough scheme the relations between Doppler effect, Hubble effect, and Hubble law could be represented as follows:

	Relative motion characterized by		Doppler effect characterized as		Hubble effect characterized as		Hubble law characterized
CM	constant relative velocity	$\Rightarrow$	corollary	$\rightarrow$	corollary	$\Leftarrow$	by definition
SRT	constant relative velocity	$\Rightarrow$	corollary	$\rightarrow$	corollary	$\Leftarrow$	by definition
GRT	change of the metrics of space-time	$\Rightarrow$	corollary	$\rightarrow$	corollary	$\Leftarrow$	by definition of the metrics
CRT	relative velocity and relative acceleration	$\Rightarrow$	corollary	$\rightarrow$	corollary	$\leftarrow$	as corollary
		$\Downarrow$					$\Uparrow$
		$\Rightarrow$	$\longrightarrow$	$\rightarrow$	$\longrightarrow$	$\rightarrow$	$\Uparrow$

6. Since the Doppler effect and the Hubble effect as kinematic effects could be described by different theoretical schemes and models of space-time the rich mathematical tools of the spaces with affine connections and metrics, considered as models of space-time, are used for description of both the effects. The aim has been to work out a theoretical model of the Doppler effect and of the Hubble

effect as corollaries only of the relative motion between emitter and observer determined by the kinematic characteristics of the relative velocity and the relative acceleration between emitter and observer from point of view of the proper frame of reference of the observer. For this task the $(n-1)+1$ formalism has been used related to the world line of an observer and its corresponding $n-1$ dimensional sub space interpreted as the observed space in the proper frame of the observer [23].

7. Our task in the present paper is to investigate the influence of the Doppler effect and of the Hubble effect on the frequency shift parameter. In section 2 the Doppler effect and shift frequency parameter are considered. In Section 3 the Hubble effect and shift frequency parameter are considered. It is shown that by the use of the shift frequency parameter, considered locally in the proper frame of reference of an observer [23], we can directly determine the radial (centrifugal, centripetal) relative velocity and acceleration as well as the tangential (Coriolis) relative velocity and acceleration. Section 4 comprises some concluding remarks. Most of the details and derivations omitted in this paper could be found in [20] and in [21].

1.1 Abbreviation and symbols

- The vector field u is the velocity vector field of an observer: $u \in T(M)$, $\dim\ M = n$, $n = 4$.
- The contravariant vector field $v_z = \mp l_{v_z} \cdot n_\perp = H \cdot l_{\xi_\perp} \cdot n_\perp = H \cdot \xi_\perp$ is orthogonal to u and collinear to $\xi_\perp$. It is called radial (centrifugal, centripetal) relative velocity.
- The function $H = H(\tau)$ is called Hubble function.
- The contravariant vector field $a_z = \mp l_{a_z} \cdot n_\perp = \overline{q} \cdot l_{\xi_\perp} \cdot n_\perp = \overline{q} \cdot \xi_\perp$ is orthogonal to u and collinear to $\xi_\perp$. It is called radial (centrifugal, centripetal) relative acceleration.
- The function $\overline{q} = \overline{q}(\tau)$ is called acceleration function (parameter).
- The contravariant vector field $v_{\eta c} = \mp l_{v_{\eta c}} \cdot m_\perp = \overline{H}_c \cdot l_{\xi_\perp} \cdot m_\perp = \overline{H}_c \cdot \eta_\perp$ is orthogonal to u and to $\xi_\perp$. It is called tangential (Coriolis) relative velocity.
- The function $\overline{H}_c = \overline{H}_c(\tau)$ is called Coriolis Hubble function.
- The contravariant vector field $a_{\eta_c} = \mp l_{a_{\eta c}} \cdot m_\perp = \overline{q}_{\eta c} \cdot l_{\xi_\perp} \cdot m_\perp = \overline{q}_{\eta c} \cdot \eta_\perp$ is orthogonal to u and to $\xi_\perp$. It is called tangential (Coriolis) relative acceleration.
- The function $\overline{q}_{\eta c} = \overline{q}_{\eta c}(\tau)$ is called Coriolis acceleration function (parameter).

- $\overline{\omega}$ is the frequency of a signal emitted by an emitter at a time $\tau - d\tau$ of the proper time of the observer.
- ω is the frequency of a signal detected by the observer at a time τ of the proper time of the observer.
- $d\tau$ is the time interval in the proper frame of reference of the observer for which the signal propagates from the emitter to the observer (detector) at a space distance dl in the proper frame of reference of the observer.
- τ is the proper time parameter of the observer in his proper frame of reference.

2 Doppler effect and shift frequency parameter

It has been shown [20], [21] that in a $(\overline{L}_n, g)$-space longitudinal (standard) and transversal Doppler effects could appear when signals are propagating from an emitter to an observer (detector) moving relatively to each other.

2.0.1 Longitudinal (standard) Doppler effect and the shift frequency parameter

1. Let a signal with frequency $\overline{\omega}$ be emitted by an emitter [20] at the time $\tau - d\tau$ and be received by an observer (detector) at the time τ.

Since $\overline{\omega} = \omega(\tau - d\tau)$ and $\omega = \omega(\tau)$, we can expand $\overline{\omega}$ in a Taylor row up to the second order of $d\tau$

$$\overline{\omega} = \omega(\tau - d\tau) \approx \omega(\tau) - \frac{d\omega}{d\tau}_{\mid\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\omega}{d\tau^2}_{\mid\tau} \cdot d\tau^2 + O(d\tau) \quad .$$

Then

$$d\omega = \overline{\omega} - \omega \approx -\frac{d\omega}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\omega}{d\tau^2} \cdot d\tau^2 \quad ,$$

$$\frac{d\omega}{\omega} = \frac{\overline{\omega} - \omega}{\omega} \approx -\frac{1}{\omega} \cdot \frac{d\omega}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{1}{\omega} \cdot \frac{d^2\omega}{d\tau^2} \cdot d\tau^2 \quad .$$

On the other side, the results in the case of a general motion of the observer read [20], [21]

$$\begin{aligned} \frac{\overline{\omega} - \omega}{\omega} &= \frac{d\omega}{\omega} = \\ &= -\frac{dl}{l_u} \cdot \left(\frac{l_{v_z}}{l_{\xi_\perp}} + \frac{l_{(a_\perp)_z}}{l_u}\right) + \frac{1}{2} \cdot \frac{dl^2}{l_u^2} \cdot \left(\frac{l_{a_z}}{l_{\xi_\perp}} + \frac{l_{(\nabla_u a)_{\perp z}}}{l_u}\right) \quad , \\ dl &= l_u \cdot d\tau \quad . \end{aligned}$$

Since

$$\frac{dl}{l_u} = d\tau \quad , \qquad \frac{dl^2}{l_u^2} = d\tau^2 \quad ,$$

we obtain the relation

$$\overline{\omega} = \omega \cdot [1 - (\frac{l_{v_z}}{l_{\xi_\perp}} + \frac{l_{(a_\perp)_z}}{l_u}) \cdot d\tau + \frac{1}{2} \cdot (\frac{l_{a_z}}{l_{\xi_\perp}} + \frac{l_{(\nabla_u a)_{\perp z}}}{l_u}) \cdot d\tau^2)] \quad .$$

2. If we consider only infinitesimal changes of the frequency ω for the time interval $d\tau$, i.e. if $d\omega = \overline{\omega} - \omega$, we can express the shift parameter $z = (\overline{\omega} - \omega)/\omega$ as an infinitesimal quantity

$$\begin{aligned} z &= \frac{d\omega}{\omega} = d(\log\ \omega) = \\ &= d\overline{z} = -\frac{1}{\omega} \cdot \frac{d\omega}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{1}{\omega} \cdot \frac{d^2\omega}{d\tau^2} \cdot d\tau^2 = \\ &= -(\frac{l_{v_z}}{l_{\xi_\perp}} + \frac{l_{(a_\perp)_z}}{l_u}) \cdot d\tau + \frac{1}{2} \cdot (\frac{l_{a_z}}{l_{\xi_\perp}} + \frac{l_{(\nabla_u a)_{\perp z}}}{l_u}) \cdot d\tau^2 \quad . \end{aligned}$$

On the other side, $d\overline{z}$ could be considered as a differential of the function $\overline{z}$ depending on the proper time τ of the observer, i.e. $\overline{z} = \overline{z}(\tau)$. It is assumed that $\overline{z}(\tau)$ has the necessary differentiability properties. The function $\overline{z}$ at the point $\overline{z}(\tau - d\tau)$ could be represented in Taylor row as

$$\overline{z}(\tau - d\tau) = \overline{z}(\tau) - \frac{d\overline{z}}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}}{d\tau^2} \cdot d\tau^2 + O(d\tau) \quad .$$

Then $\overline{z}(\tau - d\tau)$ and $d\overline{z} = \overline{z}(\tau - d\tau) - \overline{z}(\tau)$ could be written up to the second order of $d\tau$ respectively as

$$\begin{aligned} \overline{z}(\tau - d\tau) &= \overline{z}(\tau) - \frac{d\overline{z}}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}}{d\tau^2} \cdot d\tau^2 \quad , \\ d\overline{z} &= \overline{z}(\tau - d\tau) - \overline{z}(\tau) = \\ &= -\frac{d\overline{z}}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}}{d\tau^2} \cdot d\tau^2 = \\ &= -\frac{1}{\omega} \cdot \frac{d\omega}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{1}{\omega} \cdot \frac{d^2\omega}{d\tau^2} \cdot d\tau^2 = \\ &= -(\frac{l_{v_z}}{l_{\xi_\perp}} + \frac{l_{(a_\perp)_z}}{l_u}) \cdot d\tau + \frac{1}{2} \cdot (\frac{l_{a_z}}{l_{\xi_\perp}} + \frac{l_{(\nabla_u a)_{\perp z}}}{l_u}) \cdot d\tau^2 \quad . \end{aligned}$$

The comparison of the coefficients before $d\tau$ and $d\tau^2$ in the last (above) two expressions leads to the relations

$$\frac{d\overline{z}}{d\tau} = \frac{1}{\omega} \cdot \frac{d\omega}{d\tau} \quad , \qquad \frac{d^2\overline{z}}{d\tau^2} = \frac{1}{\omega} \cdot \frac{d^2\omega}{d\tau^2} \quad ,$$

$$\frac{d\overline{z}}{d\tau} = \frac{l_{v_z}}{l_{\xi_\perp}} + \frac{l_{(a_\perp)_z}}{l_u} \quad , \qquad \frac{d^2\overline{z}}{d\tau^2} = \frac{l_{a_z}}{l_{\xi_\perp}} + \frac{l_{(\nabla_u a)_{\perp z}}}{l_u} \quad .$$

The vector $\xi_\perp$ could be chosen as a unit vector, i.e. $l_{\xi_\perp} = 1$, equal to the vector $n_\perp$ showing the direction to the emitter from point of view of the observer. Then

$$\frac{d\overline{z}}{d\tau} = l_{v_z} + \frac{l_{(a_\perp)_z}}{l_u} \qquad , \qquad \frac{d^2\overline{z}}{d\tau^2} = l_{a_z} + \frac{l_{(\nabla_u a)_{\perp z}}}{l_u} \qquad .$$

Therefore, if we can measure the change (variation) of the shift frequency parameter $d\overline{z}$ in a time interval $d\tau$ we can find the centrifugal (centripetal) relative velocity and radial (centrifugal, centripetal) relative acceleration of the emitter with respect to the observer. The above relations appear as direct way for a check-up of the considered theoretical scheme of the propagation of signals in spaces with affine connections and metrics. On the other side, the explicit form of l_{v_z} and l_{a_z} as functions of the kinematic characteristics of the relative velocity and relative acceleration could lead to conclusions of the properties of the space-time model used for description of the physical phenomena. The same relations could lead to more precise assessment of the Hubble function H and the acceleration function $\overline{q}$ at a given time.

3. Let us now consider the shift frequency parameter when the observer's world line is an auto-parallel trajectory, i.e. when the velocity vector u of the observer fulfills the equation

$$\nabla_u u = f \cdot .u \qquad , \qquad f \in C^\infty(M) \ .$$

Then, because of $\overline{g}[h_u(u)] = 0$,

$$a_\perp = \overline{g}[h_u(a)] = f \cdot \overline{g}[h_u(u)] = 0 \qquad ,$$

$$\begin{aligned}(\nabla_u a)_\perp &= \overline{g}[h_u(\nabla_u(f \cdot u))] = \overline{g}[h_u((uf) \cdot u + f \cdot a)] = \\ &= (uf) \cdot \overline{g}[h_u(u)] + f \cdot \overline{g}[h_u(a)] = 0 \qquad .\end{aligned}$$

In this case, the relations are fulfilled

$$\frac{d\overline{z}}{d\tau} = \frac{l_{v_z}}{l_{\xi_\perp}} \qquad , \qquad \frac{d^2\overline{z}}{d\tau^2} = \frac{l_{a_z}}{l_{\xi_\perp}} \quad .$$

$$\frac{d\overline{z}}{d\tau} = l_{v_z} \qquad , \qquad \frac{d^2\overline{z}}{d\tau^2} = l_{a_z} \quad , \quad l_{\xi_\perp} = 1 \quad .$$

Therefore, if we can measure the change (variation) of the shift frequency parameter $d\overline{z}_c$ in a time interval $d\tau$ we can find the radial (centrifugal, centripetal) relative velocity and the radial (centrifugal, centripetal) relative acceleration of the emitter with respect to the observer moving on an auto-parallel world line.

2.0.2 Transversal Doppler effect and the shift frequency parameter

In analogous way we can find the relations between the absolute values of the tangential (Coriolis) relative velocity and the tangential (Coriolis) relative acceleration and the shift frequency parameter.

1. The relation between the frequency $\overline{\omega}$ of the emitted signals and the frequency ω of the detected signals reads [20], [21]

$$\overline{\omega} = \omega \cdot [1 - \frac{dl}{l_u} \cdot (\frac{1}{l_{\xi_\perp}} \cdot l_{v_{\eta c}} + \frac{1}{l_u} \cdot l_{(a_\perp)_c}) + \frac{1}{2} \cdot \frac{dl^2}{l_u^2} \cdot (\frac{l_{a_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(\nabla_u a)_{\perp c}})] =$$
$$= \omega \cdot [1 - (\frac{l_{v_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(a_\perp)_c}) \cdot d\tau + \frac{1}{2} \cdot (\frac{l_{a_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(\nabla_u a)_{\perp c}}) \cdot d\tau^2] \quad .$$

The shift frequency parameter has the form

$$\begin{aligned} z_c = \frac{\overline{\omega} - \omega}{\omega} &= -(\frac{l_{v_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(a_\perp)_c}) \cdot d\tau + \frac{1}{2} \cdot (\frac{l_{a_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(\nabla_u a)_{\perp c}}) \cdot d\tau^2 = \\ &= d\overline{z}_c = \\ &= -\frac{d\overline{z}_c}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}_c}{d\tau^2} \cdot d\tau^2 + O(d\tau) \approx \\ &\approx -\frac{d\overline{z}_c}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}_c}{d\tau^2} \cdot d\tau^2 \ . \end{aligned}$$

The comparison of the coefficients before $d\tau$ and $d\tau^2$ in the two expressions

$$\begin{aligned} z_c = \frac{\overline{\omega} - \omega}{\omega} &= -(\frac{l_{v_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(a_\perp)_c}) \cdot d\tau + \frac{1}{2} \cdot (\frac{l_{a_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(\nabla_u a)_{\perp c}}) \cdot d\tau^2 = \\ &= d\overline{z}_c = \\ &\approx -\frac{d\overline{z}_c}{d\tau} \cdot d\tau + \frac{1}{2} \cdot \frac{d^2\overline{z}_c}{d\tau^2} \cdot d\tau^2 \end{aligned}$$

leads to the relations

$$\frac{d\overline{z}_c}{d\tau} = \frac{l_{v_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(a_\perp)_c} \quad , \quad \frac{d^2\overline{z}_c}{d\tau^2} = \frac{l_{a_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(\nabla_u a)_{\perp c}} \quad .$$

The vector $\xi_\perp$ could be chosen as a unit vector, i.e. $l_{\xi_\perp} = 1$, equal to the vector $n_\perp$ showing the direction to the emitter from point of view of the observer. Then

$$\frac{d\overline{z}_c}{d\tau} = l_{v_{\eta c}} + \frac{1}{l_u} \cdot l_{(a_\perp)_c} \quad , \quad \frac{d^2\overline{z}_c}{d\tau^2} = l_{a_{\eta c}} + \frac{1}{l_u} \cdot l_{(\nabla_u a)_{\perp c}} \quad .$$

Therefore, if we can measure the change (variation) of the shift frequency parameter $d\overline{z}_c$ in a time interval $d\tau$ we can find the tangential (Coriolis) relative velocity and the tangential (Coriolis) relative acceleration of the emitter with respect to the observer. The above relations appear as direct way for a check-up of the considered theoretical scheme of the propagation of signals in spaces with affine connections and metrics. On the other side, the explicit form of $l_{v_{\eta c}}$ and $l_{a_{\eta c}}$ as functions of the kinematic characteristics of the relative velocity and relative acceleration could lead to conclusions of the properties of the space-time model used for description of the physical phenomena. The same relations could

lead to more precise assessment of the Hubble function H_c and the acceleration function $\overline{q}_{\eta c}$ at a given time.

2. In the case when the observer's world line is an auto-parallel trajectory, the relations are fulfilled

$$\frac{d\overline{z}_c}{d\tau} = \frac{l_{v_{\eta c}}}{l_{\xi_\perp}} \quad , \quad \frac{d^2\overline{z}_c}{d\tau^2} = \frac{l_{a_{\eta c}}}{l_{\xi_\perp}} \quad .$$

$$\frac{d\overline{z}_c}{d\tau} = l_{v_{\eta c}} \quad , \quad \frac{d^2\overline{z}_c}{d\tau^2} = l_{a_{\eta c}} \quad , \quad l_{\xi_\perp} = 1 \quad .$$

Therefore, if we can measure the change (variation) of the shift frequency parameter $d\overline{z}_c$ in a time interval $d\tau$ we can find the tangential (Coriolis) relative velocity and the tangential (Coriolis) relative acceleration of the emitter with respect to the observer moving on an auto-parallel world line.

3 Hubble effect and shift frequency parameter

It has been shown [20], [21] that in a $(\overline{L}_n, g)$-space longitudinal (standard) and transversal Hubble effects could appear when signals are propagating from an emitter to an observer (detector) moving relatively to each other.

3.1 Longitudinal (standard) Hubble effect and the shift frequency parameter

1. By the use of the relations [20], [21]

$$l_{v_z} = \mp H \cdot l_{\xi_\perp} \quad , \quad l_{a_z} = \mp \overline{q} \cdot l_{\xi_\perp} \quad ,$$

$$\frac{d\overline{z}}{d\tau} = \frac{l_{v_z}}{l_{\xi_\perp}} + \frac{l_{(a_\perp)_z}}{l_u} \quad , \quad \frac{d^2\overline{z}}{d\tau^2} = \frac{l_{a_z}}{l_{\xi_\perp}} + \frac{l_{(\nabla_u a)_{\perp z}}}{l_u} \quad ,$$

$$l_{(a_\perp)_z} = g(a_\perp, n_\perp) \quad , \quad l_{(\nabla_u a)_{\perp z}} = g(n_\perp, (\nabla_u a)_\perp) \quad ,$$

we can find the Hubble function H and the acceleration function (parameter) $\overline{q}$ respectively as

$$\frac{d\overline{z}}{d\tau} = \mp H + \frac{l_{(a_\perp)_z}}{l_u} \quad , \quad \frac{d^2\overline{z}}{d\tau^2} = \mp\overline{q} + \frac{l_{(\nabla_u a)_{\perp z}}}{l_u} \quad ,$$

$$\frac{d\overline{z}}{d\tau} = \mp H + \frac{g(a_\perp, n_\perp)}{l_u} = \mp\widetilde{H} \quad , \quad \frac{d^2\overline{z}}{d\tau^2} = \mp\overline{q} + \frac{g(n_\perp, (\nabla_u a)_\perp)}{l_u} = \mp\widetilde{q} \quad ,$$

$$\mp\widetilde{H} = \mp H + \frac{g(a_\perp, n_\perp)}{l_u} \quad , \quad \mp\widetilde{q} = \mp\overline{q} + \frac{g(n_\perp, (\nabla_u a)_\perp)}{l_u} \quad .$$

2. If the world line of the observer is an auto-parallel trajectory then we can find a direct relation between the change of the shift frequency parameter and the Hubble function H as well as the acceleration parameter $\overline{q}$

$$\frac{d\overline{z}}{d\tau} = \mp H \quad , \quad \frac{d^2\overline{z}}{d\tau^2} = \mp\overline{q} \quad .$$

3.2 Transversal Hubble effect and the shift frequency parameter

1. By the use of the relations [20], [21]

$$l_{v_{\eta c}} = \mp \overline{H}_c \cdot l_{\xi_\perp} \quad , \qquad l_{a_{\eta c}} = \mp \overline{q}_{\eta c} \cdot l_{\xi_\perp} \quad ,$$

$$\frac{d\overline{z}_c}{d\tau} = \frac{l_{v_{\eta c}}}{l_{\xi_\perp}} + \frac{1}{l_u} \cdot l_{(a_\perp)_c} \quad , \qquad \frac{d^2\overline{z}_c}{d\tau^2} = \frac{d^2\overline{z}_c}{d\tau^2} = l_{a_{\eta c}} + \frac{1}{l_u} \cdot l_{(\nabla_u a)_{\perp c}} \quad ,$$

$$l_{(a_\perp)_c} = g(a_\perp, m_\perp) \quad , \qquad l_{(\nabla_u a)_{\perp c}} = g(m_\perp, (\nabla_u a)_\perp) \quad ,$$

we can find the transversal Hubble function H_c and the transversal acceleration function (parameter) $\overline{q}_{\eta c}$ respectively as

$$\frac{d\overline{z}_c}{d\tau} = \mp \overline{H}_c + \frac{1}{l_u} \cdot l_{(a_\perp)_c} \quad , \qquad \frac{d^2\overline{z}_c}{d\tau^2} = \mp \overline{q}_{\eta c} + \frac{1}{l_u} \cdot l_{(\nabla_u a)_{\perp c}} \quad ,$$

$$\frac{d\overline{z}_c}{d\tau} = \mp \overline{H}_c + \frac{1}{l_u} \cdot g(a_\perp, m_\perp) = \mp \widetilde{H}_c \quad ,$$

$$\frac{d^2\overline{z}_c}{d\tau^2} = \mp \overline{q}_{\eta c} + \frac{1}{l_u} \cdot g(m_\perp, (\nabla_u a)_\perp) = \mp \widetilde{q}_{\eta c} \quad ,$$

$$\mp \widetilde{H}_c = \mp \overline{H}_c + \frac{1}{l_u} \cdot g(a_\perp, m_\perp) \quad , \qquad \mp \widetilde{q}_{\eta c} = \mp \overline{q}_{\eta c} + \frac{1}{l_u} \cdot g(m_\perp, (\nabla_u a)_\perp) \quad ,$$

Therefore, we have fine 4 equations for the vector field u as space-time velocity of the observer if the first and second derivative of $\overline{z}$ and $\overline{z}_c$ are found by the experiments.

$$\frac{d\overline{z}}{d\tau} = \mp H + \frac{g(a_\perp, n_\perp)}{l_u} \quad ,$$

$$\frac{d^2\overline{z}}{d\tau^2} = \mp \overline{q} + \frac{g(n_\perp, (\nabla_u a)_\perp)}{l_u} \quad ,$$

$$\frac{d\overline{z}_c}{d\tau} = \mp \overline{H}_c + \frac{1}{l_u} \cdot g(a_\perp, m_\perp) \quad ,$$

$$\frac{d^2\overline{z}_c}{d\tau^2} = \mp \overline{q}_{\eta c} + \frac{1}{l_u} \cdot g(m_\perp, (\nabla_u a)_\perp) \quad ,$$

where

$$
\begin{aligned}
H &= \frac{1}{n-1} \cdot \theta \mp \sigma(n_\perp, n_\perp) \quad , \\
\overline{H}_c &= \frac{1}{n-1} \cdot \theta \mp \sigma(m_\perp, m_\perp) \quad , \\
\overline{q} &= \frac{1}{n-1} \cdot U \mp \ {}_sD(n_\perp, n_\perp) \ , \\
\overline{q}_{\eta c} &= \mp \ {}_sD(m_\perp, m_\perp) \quad , \\
\pm l_u \cdot \frac{dl_u}{d\tau} &= g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u) \quad .
\end{aligned}
$$

2. If the world line of the observer is an auto-parallel trajectory then we can find a direct relation between the change of the shift frequency parameter and the Hubble function $\overline{H}_c$ as well as the acceleration parameter $\overline{q}_{\eta c}$

$$
\frac{d\overline{z}_c}{d\tau} = \mp \overline{H}_c \qquad , \qquad \frac{d^2 \overline{z}_c}{d\tau^2} = \mp \overline{q}_{\eta c} \quad .
$$

3. The finding out of u by the use of the 4-equations is based on the fact that the quantities H, H_c, $\overline{q}$, $\overline{q}_{\eta c}$, $g(n_\perp, a_\perp)$, $g(n_\perp, (\nabla_u a)_\perp)$, $g(a_\perp, m_\perp)$, and $g(m_\perp, (\nabla_u a)_\perp)$ are functions of the vector field u and its covariant derivatives. If the affine connections and metrics are given in a $(\overline{L}_n, g)$-space then the solving of the above equations could determine the space-time velocity of the observer with respect to the emitter.

4 Conclusion

In the present paper we have considered the Doppler effect and the Hubble effect in a $(\overline{L}_n, g)$-space and their relations to the shift frequency parameters corresponding to the longitudinal and transversal effects. It is shown that these effects lead to direct check-up of the theoretical scheme and could be used for finding out the relative radial (centrifugal, centripetal) velocities and accelerations as well as the relative tangential (transversal, Coriolis) velocities and accelerations of moving astronomical objects from point of view of the proper frame of reference of an observer (detector).

The Doppler effects and the Hubble effects are considered on the grounds of purely kinematic considerations. It should be stressed that the Hubble functions H and $\overline{H}_c$ are introduced on a purely kinematic basis related to the notions of relative radial (centrifugal, centripetal) velocity and to the notions of tangential (Coriolis) velocities respectively. They could be found directly by the use of the measurements of the shift frequency parameters. It should be noted that $\overline{H}_c$ does not exists in the Einstein theory of gravitation. The dynamic interpretations of H and $\overline{H}_c$ in a theory of gravitation depend on the structures of the theory and the relations between the field equations and on both the functions. In this paper, it is shown that notions the specialists use to apply in theories

of gravitation and cosmological models could have a good kinematic grounds independent of any concrete classical field theory. Doppler effects, and Hubble effects could be used in mechanics of continuous media and in other classical field theories in the same way as the standard Doppler effect is used in classical and special relativistic mechanics.

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